OPTIMIZATION OF A MARINE MHD PROPELLER

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UDC 538.4:629.12

The literature of recent years contains several papers [1-3] on the design of conduction-type MHD propellers for sea water. However, these do not deal with an important aspect of the performance of an MHD propeller, namely modes with the highest efficiency.

Here we consider the basic physical processors in a conduction MHD propeller and define the optimum modes of operation, and we give results from engineering calculations on the efficiency of such a propeller, which show that such MHD machines are highly efficient.

A marine MHD propeller is the name given to a direct-flow hydroreactive MHD motor built to drive transportation which uses seawater as the electrically conducting body.

Seawater has an electrical conductivity of $\sigma \sim 4\text{-}6\ (\Omega\cdot\mathrm{m})^{-1}$, i.e., less by a factor 10^5-10^6 than that of a liquid metal, so only a conduction-type MHD motor can be used in an MHD propeller.

In what follows, we use the term MHD motor if the essence of the processes is dependent only on the effects in the channel, while an MHD propeller is used if we consider the conditions of displacement.

A basic parameter of reactive MHD motors is the thrust, by which is meant the principal vector of the bulk and surface forces acting on elements of the construction from the external and internal flows (Fig. 1).

The thrust of an MHD motor defined in this way is termed effective and can be written as

$$\mathbf{R}_0 = -\int\limits_{V^*} \mathbf{j} \times \mathbf{B} dV - \int\limits_{\Sigma^*} \mathbf{p}_n d\Sigma - \int\limits_{\Sigma_*} \boldsymbol{\pi}_n d\Sigma, \tag{1}$$

where Σ^* and Σ_* are correspondingly the internal and external contours of the MHD motor, V^* is the working volume of the channel, which is bounded by the surface Σ^* and the sections a–a and 2–2.

In what follows it is assumed that the magnetic fluxes are closed within the volume $V^* + V_*$, where V_* is the volume of the MHD motor bounded by the surfaces Σ^* and Σ_*.

In (1) the symbols are as follows: $\mathbf{p}_n$ and $\boldsymbol{\pi}_n$ are the density of the surface forces in the internal and external flows, $\mathbf{j}$ is current density, $\mathbf{B}$ is magnetic field induction, and $\mathbf{n}$ exterior normal to the corresponding surface.

According to Newton's third law, $-\mathbf{R}_0$ is the principal vector of the bulk and surface forces acting from the motor on the external and internal flows.

In (1) there are also the external resistance forces, which are dependent on the attachment of the MHD motor to the transport object. For this reason one considers an isolated motor [1, 3], in which these resistance forces are zero. For this purpose we assume that the external flow around the motor is reversible, while the thrust of the isolated motor is termed internal.

We derive a working relation for the internal thrust $\mathbf{R}$ of an MHD motor. For this purpose we apply a momentum equation [4] in the steady state for the volume of seawater V (Fig. 1) bounded by the surface of the current tube $\Sigma = \Sigma_0 + \Sigma^* + S_1 + S_2$,

$$\int\limits_{\Sigma} \rho v_n \mathbf{v} d\Sigma = \int\limits_{V} \mathbf{j} \times \mathbf{B} dV + \int\limits_{\Sigma} \mathbf{p}_n d\Sigma, \tag{2}$$

where ρ is the density of seawater.

Orenburg and Perm'. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 86-94, May-June, 1981. Original article submitted March 13, 1980.

 0021-8944/81/2203-0364$07.50

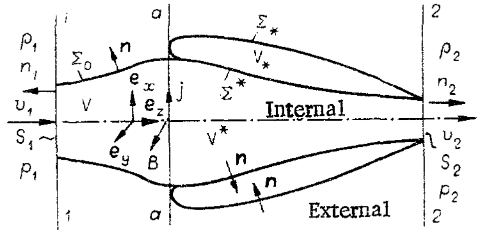

Fig. 1

We assume that the flow speeds are homogeneous in the sections 1–1 and 2–2 and that the internal stresses amount to pressures, i.e., $\mathbf{p}_n = -p\mathbf{n}$, while in accordance with the above assumptions $\int\limits_{V-V^*} \mathbf{j} \times \mathbf{B} dV = 0$, and for the external flow we have $\pi_n = -p_0\mathbf{n}$, where p_0 is the pressure of the surrounding medium (we neglect the change in hydrostatic pressure), and from (1) and (2) we get the desired expression for the vector of the internal thrust $\mathbf{R}$ for the MHD motor:

$$\mathbf{R} = -\int\limits_{S_1+S_2} (\mathbf{p}_n + \mathbf{n}p_0 - \rho v_n \mathbf{v})\, d\Sigma = (p_1 - p_0 + \rho_1 v_1^2)\, S_1 \frac{\mathbf{v}_1}{v_1} - (p_2 - p_0 + \rho_2 v_2^2)\, S_2 \frac{\mathbf{v}_2}{v_2}. \tag{3}$$

where $\mathbf{n}_1 = -\mathbf{v}_1/v_1$; $\mathbf{n}_2 = \mathbf{v}_2/v_2$.

We now use the fact that the pressure in the unperturbed incident flow is $p_1 = p_0$ and the fact that the pressure in the jet at the exit from the nozzle is $p_2 = p_0$, and from (3) we get a formula for the thrust of the MHD motor [1-4]

$$R = G(v_2 - v_1), \tag{4}$$

where $G = \rho_1 v_1 S_1 = \rho_2 v_2 S_2$ is the mass flow rate of seawater through the channel and v_1 and v_2 are respectively the speed of the unperturbed incident flow (speed of the transportation) and speed of the jet at the exit from the nozzle relative to the coordinate system rigidly coupled to the MHD motor.

We now consider the parameters of the energy performance of the propulsion system. The overall efficiency of the MHD motor is determined by the relation of the thrust power to the input electrical power

$$\eta_0 = Rv_1 \left(\int\limits_{V_+} \mathbf{jE} dV \right)^{-1},$$

where V_+ is the volume including the active and end zones of the MHD channel.

The propulsive or thrust mechanical efficiency for an MHD motor is equal to the ratio of the thrust power to the mechanical power:

$$\eta_1 = Rv_1 \left(\int\limits_{\Sigma} \rho \frac{v^2}{2} v_n d\Sigma \right)^{-1} = Rv_1 \left[G \left(\frac{v_2^2}{2} - \frac{v_1^2}{2} \right) \right]^{-1} = \frac{2}{1 + v_2/v_1}.$$

Here we only use (4) and the condition for equality of the pressures in the sections S_1 and S_2.

The electrical efficiency of the motor is the ratio of the electromagnetic power in the active zone of the channel to the input electrical power:

$$\eta_2' = \int\limits_{V_a} \mathbf{v} (\mathbf{j} \times \mathbf{B})\, dV \left(\int\limits_{V_+} \mathbf{jE} dV \right)^{-1}, \tag{5}$$

where V_a is the volume of the active zone.

The hydraulic efficiency is equal to the ratio of the mechanical power to the electromagnetic, i.e.,

$$\eta_3 = \int_{\Sigma} \rho \frac{v^2}{2} v_n d\Sigma \left(\int_{V_a} \mathbf{v} (\mathbf{j} \times \mathbf{B})\, dV \right)^{-1}.$$

Clearly we have

$$\eta_0 = \eta_1 \eta_2' \eta_3. \tag{6}$$

The hydraulic efficiency characterizes the perfection of the flow part in the motor and will be used in the form

$$\eta_3 = \frac{v_1 + v_2}{\langle v \rangle} R \left(\int_{V_a} \mathbf{e}_z (\mathbf{j} \times \mathbf{B})\, dV \right)^{-1}, \tag{7}$$

where <v> is the mean velocity of the seawater in the channel, which is defined by

$$\int_{V_a} \mathbf{v} (\mathbf{j} \times \mathbf{B})\, dV = \langle v \rangle \int_{V_a} \mathbf{e}_z (\mathbf{j} \times \mathbf{B})\, dV.$$

The electrical efficiency η_2' incorporates various causes of energy loss: Joule dissipation in the active and end zones of the channel [5] and additional Joule dissipation in the two-phase bubble layers at the electrodes due to electrolysis of the seawater, which leads to chemical polarization.

We consider an ideal marine MHD motor in which there is no electrolysis of the seawater or electrical energy dissipation outside V_a, which enables one to consider the sources of loss separately.

For this purpose we write the electrical efficiency of (5) as the product

$$\eta_2' = \frac{\int_{V_a} \mathbf{v} (\mathbf{j} \times \mathbf{B})\, dV}{\int_{V_a} (\mathbf{jE})_0\, dV} \cdot \frac{\int_{V_a} (\mathbf{jE})_0\, dV}{\int_{V_a} \mathbf{jE}\, dV} \cdot \frac{\int_{V_a} \mathbf{jE}\, dV}{\int_{V_+} \mathbf{jE}\, dV}, \tag{8}$$

where

$$\eta_2 = \int_{V_a} \mathbf{v} (\mathbf{j} \times \mathbf{B})\, dV \left(\int_{V_a} (\mathbf{jE})_0\, dV \right)^{-1} \tag{9}$$

is the electrical efficiency of an ideal marine motor, while $(\mathbf{jE})_0$ is the density of the electrical power in the channel. The second cofactor in (8) is

$$\eta_5 = \int_{V_a} (\mathbf{jE})_0\, dV \left(\int_{V_a} \mathbf{jE}\, dV \right)^{-1} \tag{10}$$

and incorporates the loss of electrical power in the active zone of the channel due to gas production at the electrodes, which may be called the electrochemical efficiency of the motor.

The third cofactor in (8) is

$$\eta_4 = \int_{V_a} \mathbf{jE}\, dV \left(\int_{V_+} \mathbf{jE}\, dV \right)^{-1} \tag{11}$$

and reflects the effects of Joule dissipation in the end zones of the channel.

The overall efficiency η_0 is given by (6) and (8)-(11) as the product

$$\eta_0 = \eta_1 \eta_2 \eta_3 \eta_4 \eta_5 .$$

One can compare the performance of an MHD motor with that of other types of propulsion system, such as a propeller screw, by reference to

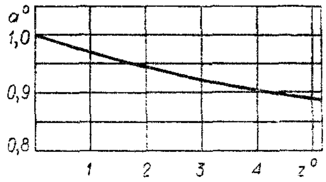

Fig. 2

$$\eta = \frac{\eta_0}{\eta_1} = \eta_2\eta_3\eta_4\eta_5, \tag{12}$$

which may be called the overall efficiency or internal efficiency of the MHD motor.

The literature carries [3] two engineering schemes for MHD motors: with isovelocity and with isobaric MHD channels. These schemes allow one to estimate the limiting performance of a reactive MHD motor as regards efficiency and specific thrust.

The isobaric form has some advantages over the isovelocity one, including lack of load on the walls of the channel, ease in eliminating the bubble layers at the electrodes, and absence of gas bubble repulsion [6-8] against the main flow, which in an isovelocity channel can sometimes lead to instability in the operation of the MHD motor.

We consider an isobaric motor with channel length h. Let the distance between the electrodes be $a(z)$, where z is the longitudinal coordinate in the channel, while the distance between the insulating walls is l = const. It is of interest to estimate approximately the profile of the channel that provides $\partial p/\partial z = 0$ with $B(z) = B$ = const, $Re_m \ll 1$, and ρ = const.

We consider the momentum equation in the hydraulic approximation

$$\rho v \frac{dv}{dz} = j(z) B - \frac{\lambda_m}{D} \rho \frac{v^2}{2},$$

where λ_m is the hydraulic-friction coefficient and D is the hydraulic diameter of the channel, together with the expression for the density of the electromagnetic force

$$j(z) B = \frac{\sigma}{a(z)} (U - a_1 v_1 B) B,$$

where U is the potential difference between the electrodes and $a_1 = a(0)$; $v_1 = v(0)$, where we have put that $a(0) - a(h) \ll a(h)$, together with the conditions for continuity in the flow and for uniqueness, which gives

$$z^0 = -\frac{1}{f^0} \frac{1}{\alpha - \beta} \ln\left| \frac{1-\beta}{a^0 - \beta} \frac{a^0 - \alpha}{1-\alpha} \right|, \tag{13}$$

from which one can determine the profile for an isobaric motor. The symbols in (13) are

$$z^0 = \frac{z}{a_1}, \quad f^0 = (\rho v_1^2)^{-1} \sigma (U - a_1 v_1 B) B, \quad a^0 = \frac{a(z)}{a_1},$$

while α and β are the roots of the equation

$$(a^0)^2 - \frac{\lambda_m}{4} \frac{m}{f^0} a^0 - \frac{\lambda_m}{4 f^0} = 0,$$

where $m = a_1/l$.

One has to establish the sufficiency of the hydraulic approximation in calculating an MHD propeller. In this connection it should be noted that MHD propellers are proposed for high-speed transportation and high thrust levels. Then the liquid moves in the channel with large Reynolds numbers (~10^7-10^8), and then the maximum velocity in any cross section differs little from the mean.

Figure 2 shows the profile of an isobaric channel calculated from (13) with the following initial data: v_1 = 20 m/sec, $\lambda_m = 7.13 \cdot 10^{-3}$, a_1 = 3.85 m, $U \simeq 10^3$ V, $\sigma = 5\ (\Omega \cdot m)^{-1}$,

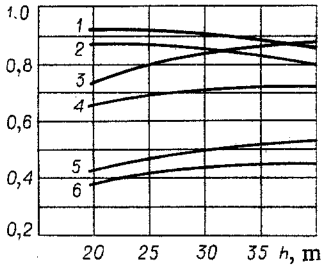

Fig. 3

B = 10 T, m = 4. The graph shows that the curvature of the liquid current lines in the channel is small, and also that the angle they form is small, i.e., the flow in the channel is smoothly varying and may be considered as one-dimensional.

Some interest attaches to the mode of operation of an MHD motor with the maximum η. This condition can be found from the following problem: We are given the physical constants of seawater σ and ρ and given quantities R, v_1, m, B, and h from which to determine the geometry of the channel, the electrical parameters, and the performance parameters such that η is largest.

To define this state we consider a quantity Θ, which complements the electrical efficiency of an ideal motor η_2 to one, i.e.,

$$\eta_2 + \Theta = 1. \tag{14}$$

In the pumping state in an MHD conduction machine $\Theta = s > 0$ (slip), while in the generator state $-\Theta = s/(s-1)$, $s < 0$; the convenience of the expression lies in simplifying the expressions for the motor in the generator state, e.g., when the vessel is slowing down.

In an isobaric channel without inlet and outlet tubes we have $\langle v \rangle \approx 0.5(v_1 + v_2)$; then from (7) we have that η_3 can be put as

$$\eta_3 = \frac{\rho v_1}{\sigma h B^2} \frac{1-\Theta}{\Theta}(k-1), \tag{15}$$

where $k = v_2/v_1$ is determined from the magnetohydrodynamic modification of Bernoulli's equation

$$k = \frac{4-\zeta_\Sigma}{4+\zeta_\Sigma} + \frac{4}{4+\zeta_\Sigma} \frac{\sigma h B^2}{\rho v_1} \frac{\Theta}{1-\Theta}.$$

Here ζ_Σ is the total hydraulic-resistance coefficient for the channel, which goes with the ordinary hydraulic resistances in the flow part to incorporate the loss of mechanical power in the jet due to the entry and exit from the zone of homogeneous magnetic field. The coefficient ζ incorporates these losses and can be found by solving for the Joule losses in the end zones [5] as

$$(1/2)\zeta = \psi(\infty)\sigma a B^2/\rho v, \tag{16}$$

where a is the distance between the electrodes, where the velocity of the jet is considered homogeneous, while the coefficient 1/2 allows for the presence of two zones.

The energy losses due to the longitudinal end effect may be determined by solving the model problem for the effective internal resistance of the channel and calculating the geometrical function Φ [5], which is convenient to put into correspondence with the geometrical conductivity of the longitudinal end effect C from $\Phi = (h/a)(C+1)$.

The simple assumption that C is independent of the slip enables us to put (11) as

$$\eta_4 = \Theta/(\Theta + C). \tag{17}$$

The total efficiency of (12) takes the following form on the basis of (14), (15), and (17):

$$\eta = \frac{4}{4+\zeta_\Sigma} \Theta \frac{1-\Theta}{\Theta + C}\left(1 - K\frac{1-\Theta}{\Theta}\right)\eta_5, \tag{18}$$

where $K=\frac{1}{2}\frac{\zeta_{\Sigma}\rho v_1}{\sigma h B^2}$ is a parameter of the isobaric MHD motor.

The electrochemical efficiency η_5 is substantially affected by the current density j, since the bulk gas content increases with this, which reduces the effective electrical conductivity of the seawater.

If we consider the analogy between bubble boiling in forced convection and gas production at the platinum electrodes in the marine MHD motor, then from (18.4) of [9] we can get the following estimate for the limiting current density in an MHD motor using Faraday's laws of electrolysis:

$$j_{*i}=4\frac{F}{\mu_i}c_{f0}\varphi_*(1-\varphi_*)\sqrt{\rho\rho_i}w_0, \qquad (19)$$

where the subscript i denotes the anode or cathode of the MHD motor, F is Faraday number, μ_i is the molecular weight of the corresponding gas (chlorine at the anode and hydrogen at the cathode), c_{f_0} is the coefficient of friction for j = 0, φ_* is the critical gas content in the bubble flow, ρ_i is the density of the corresponding gas, w_0 is the bulk velocity of the liquid phase as assigned to the complete cross section of the channel.

The condition for closest packing of spherical bubbles in the liquid gives $\varphi_*=\pi/6$.

Calculation of j_{*i} from (19) shows that the value is greater by a factor of 10^3–10^4 than the current density expected in a real system. Therefore, one can assume that for $j \ll j_{*i}$ we have $\eta_5 \approx 1$.

Increased pressure in the environment is important in suppressing the gas production in a marine MHD motor.

It can be shown that the highest value of η of (18) is attained for

$$\Theta_{\text{opt}}=\sqrt{C^2+\frac{K(1+2C)+C}{K+1}}-C. \qquad (20)$$

A similar method can be used to find the optimum conditions for an MHD motor with an isovelocity channel. In that case we have

$$\Theta_{\text{opt}}=\sqrt{C^2+\frac{Q(1+2C)+C}{Q+1}}-C, \qquad (21)$$

where

$$Q=\frac{1}{2}\frac{\zeta_{\Sigma}\rho v}{\sigma h B^2}$$

is the parameter of the isovelocity MHD motor and v is the speed of the seawater in the channel.

Equations (20) and (21) can be used in the engineering design for reactive MHD motors. We consider this briefly on the basis of (20). We first note that the hydraulic-resistance coefficient for an MHD channel is not known at the start of the calculation and must be refined by successive approximation. In the first approximation we specify $\zeta_{\Sigma}^{(1)}$ and calculate $K^{(1)}$ for the isobaric motor. Further we assume that $C^{(1)} = 0$ in a first approximation and from (20) we determine $\Theta_{\text{opt}}^{(1)}$; from $\zeta_{\Sigma}^{(1)}, K^{(1)}, \Theta_{\text{opt}}^{(1)}$ we calculate the reduced flow speed $k^{(1)} = v_2/v_1$ from a modified form of Bernoulli's equation on the assumption that the speed of the water at the inlet to the channel is equal to the speed of the vessel v_1. From the thrust formula (4) with known $k^{(1)}$ we get the cross sectional area of the channel at the inlet

$$S_1=\frac{R}{\rho v_1^2(k-1)}.$$

The values of S_1 and m allow one to determine parameters a and l for the inlet, while the equation of continuity for l = const and $k^{(1)}$ gives us the geometry of the channel at the exit. The channel profile after refinement of all the quantities may be constructed from (13). Then empirical hydraulic formulas are used to calculate the hydraulic-resistance coefficient, the friction, the resistance coefficient of (16), and the overall resistance coefficient of the channel. We neglect the curvature of the channel in the average cross

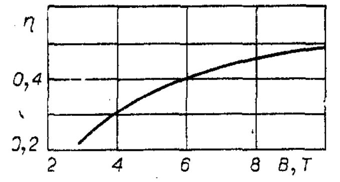

Fig. 4

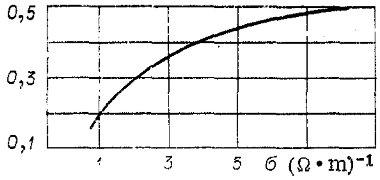

Fig. 5

section on the basis of the solution of [5] and calculate the geometircal function and the geometrical conductivity for the longitudinal effect $C^{(2)}$. Then the calculations are repeated to refine ζ_Σ and C together with all the quantities dependent on these. Practical calculations show that two or three approximations will suffice. Then from (14) we get the mean current density from Ohm's law as

$$\langle j\rangle = \frac{1}{2}(k+1)\frac{\Theta_{opt}}{1-\Theta_{opt}}\sigma v_1 B$$

and the current in the active zone of the channel as

$$I_0 = \sigma U \Theta_{opt} l \int_0^h \frac{dz}{a(z)} \approx \langle j\rangle lh.$$

From (17) we get the total current through the motor as

$$I = I_0\left(1 + \frac{C}{\Theta_{opt}}\right). \tag{22}$$

The voltage on the electrodes is determined from Θ_{opt} via

$$U = \frac{a_1 v_1 B}{1-\Theta_{opt}}. \tag{23}$$

The electrical power N required to supply the MHD propeller is given by the product of (22) and (23):

$$N = \frac{a_1 v_1 B I_0}{1-\Theta_{opt}}\left(1 + \frac{C}{\Theta_{opt}}\right).$$

The control parameters that influence Θ_{opt} are the geometry of the channel, in particular the height of the electrodes, and the electrical parameters I and U, which provide the optimum value of Θ for a motor with particular initial data: R, σ, h, v_1, m, ρ.

A different calculation is required for another set of initial data, e.g., to find the thrust for a given geometry in the optimum mode of operation, but there is no difference of principle.

Figure 3 shows the results for a marine motor with an isobaric channel. The initial data were: R = 100 T, v_1 = 20 m/sec, σ = 5 $(\Omega\cdot m)^{-1}$, B = 10 T, ρ = 10^3 kg/m^3. We varied h and calculated η_1 (curve 1), η_2—*2*, η_3—*3*, η_4—*4*, η—*5*, η_0—*6*. The formulas of [5] were used for the end losses. There were no nonconducting baffles in the channel. It is clear that η increases with the length of the channel and rises to 50%. The rise in η is due to the reduction in the end loss as h increases. It would be erroneous to assume that reduction in h reduces the size of the superconducting magnet system, because the area of the channel for a constant thrust increases as h is reduced.

Figure 4 shows η as a function of B. The initial data were as in the previous example, while h = 30 m. There is a monotone increase in the efficiency with B. Figure 4 also shows that it is undesirable to design a marine motor with B $\leqslant$ 4 T because of the low efficiency. Also, there is a substantial increase in the size of the channel for a given R as B decreases.

Figure 5 shows the effect of the conductivity on η. The initial data were: R = 100 T, h = 30 m, v_1 = 20 m/sec, v = 25 m/sec, B = 10 T, a = l = 2.32 m, and the geometry of the channel was optimum for the conductivity σ = 5 $(\Omega\cdot m)^{-1}$. Figure 5 shows that the operation of an MHD motor designed for southern seas becomes undesirable in northern seas, where the electrical conductivity is lower, on account of the marked reduction in efficiency. On the other hand, the use of MHD motors designed for northern seas in water of higher conductivity will increase the efficiency.

We have thus solved the variational problem to optimize a marine MHD motor in the hydraulic approximation for small Re_m. The efficiency calculations show that such systems are promising.

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